ISSN 2075-1087, Gyroscopy and Navigation, 2024, Vol. 15, No.2, pp. 60-77
Russian Text © The Author(s), 2024, published in Giroskopiya i Navigatsiya, 2024, No.2, pp. 98-122.

Algorithm for Determining the Coordinates and Motion Parameters of a Marine Object without Special Maneuvering of Observer

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Received: January 9, 2024; reviewed: April 10, 2024; accepted: April 10, 2024

Abstract: The paper demonstrates an automatic algorithm for determining the coordinates and motion parameters of a marine object detected by a submarine sonar system working in the passive mode. This algorithm does not require any special maneuvering of the observer. The description of the algorithm and the results of its simulation for standard situations are presented.

Keywords: underwater acoustics, marine object, measurements of object's coordinates and motion parameters, simulation.

1. INTRODUCTION

The coordinates and motion parameters (bearing *P*, distance *R*, heading *K*, and velocity *V*) (CMP) of a marine object detected by the sonar system of a submarine in the passive mode are the basis for generating the motion parameters in order to safely pass clear of it [1, 2].

All known algorithms for determining the CMP in the passive mode are based on the object's bearing change over time. However, the problem is that in case of straight-line and uniform motion of the object and observer (and this is how the marine objects do move in the absolute majority of situations), the CMP determination using the bearings only never provides an unambiguous result [1–5]. This problem is solved either by special maneuvering of the observer submarine, or by using some additional information about the heading, velocity or distance to the object observed.

As a result, all known Russian and international algorithms for determining the CMP can be conventionally divided into two groups [2]:

- automatic algorithms (N bearings, etc.) which only use the behavior of the observed object's bearings and can be implemented with mandatory special maneuvering of the observer submarine during the mission;
- automated algorithms (search method, etc.) which are based on the use of bearings behavior and additional information (usually, observed object's velocity) estimated by the sonar operator while listening the object's noise, taking into account its class.

Both algorithms are imperfect. The disadvantage of the first group of algorithms is unacceptably long time of execution. Furthermore, they cannot be used in the areas of heavy traffic because the observer submarine has to maneuver all the time. On the other hand, the second group of algorithms provides the CMP output with low accuracy. Thus, it is still an urgent task to develop an algorithm for determining the CMP, which would have no disadvantages mentioned above, and to implement it in practice [6].

The aim of this work is to describe an automatic algorithm for determining the CMP based on the data of the submarine sonar system working in the passive mode, which does not require any special maneuvering of the observer. The idea of the proposed algorithm has been presented by the authors in [7–9].

2. ALGORITHM RATIONALE

The discussed approach consists in automatic generation of estimates of the observed object's velocity, distance and heading by known methods, and in the use of these data in determining the CMP based on the behavior of the object's bearings.

When developing the algorithm, the following was taken into consideration.

1. To estimate the object's velocity and the distance to it with acceptable accuracy, it is necessary to know the class of the observed object. Since it takes some time (sometimes long enough) to determine the class of an object, the CMP determination should start from the moment it has been detected, and this task should be solved in parallel for each of the hypotheses about the class of the detected object. As a result, when the object's class is finally identified, the CMP data corresponding to this class will be selected from all the results of CMP determination obtained for various hypotheses on the object's class as of the current time point. However, it is necessary to continue determining the CMP for various hypotheses, because the decision on the object's class may change in the course of its observation.
2. It is assumed that the object's trajectory consists of tacks. The object is moving rectilinearly and uniformly on each tack. The current tack ends and the new one starts at the moment the object changes its heading and/or velocity. Therefore, the algorithm should be able to detect the moment of tack change and to start determining the CMP on the new tack from the beginning, using the output CMP from the previous tack.
3. The algorithm for determining the CMP should not set any requirements for the observer maneuvering; the CMP should be generated both when moving with a constant velocity vector (including the zero one) and when maneuvering with regard to heading and/or velocity.
4. The algorithm for determining the CMP should be synthesized based on the maximum likelihood criterion, which has proven to be quite efficient in solving similar problems [10–12].
5. The algorithm for determining the CMP should function cyclically with the period of measurements of the observed object's bearings. At the same time, the algorithm should not impose any restrictions on the periodicity and synchronicity of the input data: while the object's bearing should be measured periodically enough, its velocity and distance can be determined much less frequently or even once in a particular case.

The algorithm synthesis in accordance with the maximum likelihood criterion is based on the stochastic model of object's motion on a straight-line tack, which has the form:

$$\hat{P}_i = P\left(t_i, K, V, R_j\right) + \Delta P_i, \tag{1}$$

where $\hat{P}_i$ is the estimate of the object's bearing at the time point t_i;

$P(t_i, K, V, R_j)$ is a non-random function which relates the actual value of the object's bearing at the time point t_i and its heading K, velocity V and distance R_j at an arbitrary time point t_j;

ΔP_i is a random error of the bearing estimate $\hat{P}_i$, assumingly distributed according to the normal law with zero mathematical expectation and the standard deviation σ_P calculated by the formula [13]:

$$\sigma_P = 0{,}2 \cdot \frac{\Delta q}{Q_{ind}}, \tag{2}$$

where σ_P is RMS error of the bearing estimate, deg;

Δq is the width of antenna directional characteristic, deg;

Q_{ind} is the indicator (output) signal-to-noise ratio (SNR).

The function $P(t_i, K, V, R_j)$ is related to the motion parameters of the observed object and the observer submarine by the formula [2]:

$$P\left(t_i, K, V, R_j\right) = = arctg \frac{X_н\left(t_j\right) + R_j \cdot \sin\left(P_j\right) + V \cdot \left(t_i - t_j\right) \cdot \sin(K) - X_н\left(t_i\right)}{Y_н\left(t_j\right) + R_j \cdot cos\left(P_j\right) + V \cdot \left(t_i - t_j\right) \cdot cos(K) - Y_н\left(t_i\right)}, \quad i=1,...,N, \tag{3}$$

where $X_н(t_i)$, $Y_н(t_i)$ are Cartesian coordinates of the observer submarine, corresponding to the time point t_i;

N is the number of time points from the beginning of the current tack, at which the object's bearings were measured;

R_j, P_j are the object's distance and bearing corresponding to some arbitrary time point t_j from the array $i = 1, \ldots, N$.

If we fix the values of heading, velocity and distance of the object in the right part of (1), then the bearing measurement error ΔP_i will be the only random value. As a result, the conditional probability density function (PDF) for the bearing estimate at the time point t_j will be written as follows [14]:

$$g_{\hat{P}_i/K,V,R_j}\left(p_i\right) = g_{\Delta P_i}\left(p_i - P\left(t_i, K, V, R_j\right)\right), \tag{4}$$

where $g_{\Delta Pi}(p)$ is the PDF of bearing error in the i-th cycle of the problem solution, which is assumed to be a normal PDF with zero mathematical expectation and the RMS deviation σ_{Pi} calculated by (2).

Since the bearings are measured in different time intervals, their measurement errors are independent. Therefore, the joint conditional (depending on the object's heading, velocity and distance) PDF of the estimates of all bearings will be equal to the product of conditional PDF of estimates of individual bearings:

$$g_{\hat{P}_1,\ldots,\hat{P}_N/K,V,R_j}\left(p_1,\ldots,p_N\right)= =\prod_{i=1}^{N} g_{\Delta P_i}\left(p_i - P\left(t_i,K,V,R_j\right)\right). \quad (5)$$

One of the algorithms for determining the CMP of the detected object using only the array of bearings is to substitute the bearing estimates $\hat{P}_i$ instead of non-random arguments p_i in the right part of (5) (thereby turning (5) into a likelihood function that only depends on the object's heading, velocity and distance) and to find the values of the object's heading, velocity and distance corresponding to its global maximum. However, as was mentioned above, when using the row of bearings only, the problem cannot be solved unambiguously in case of uniform and straight-line motion of the observed object and the observer. It is necessary to involve some additional information such as the heading, velocity and distance estimates obtained from other information processing, and to form joint PDF of the estimates of the object's heading, velocity and distance. The CMP corresponding to the global maximum of such PDF will be the most plausible estimates of the CMP.

Let us begin with the estimation of a detected object's velocity. Since the object's velocity depends on its class, it is advisable to proceed as follows. Using reference books and statistical data, determine the mean value $m_{V/\omega}$ and RMS deviation $\sigma_{V/\omega}$ of velocity for the objects of each class ω that may be detected in the given navigation area. For example, in the inland seas, the most frequent classes are conventional submarines, medium to low-tonnage commercial vessels, and surface warships of corvette and frigate classes. In open oceanic areas, nuclear-powered submarines, large commercial vessels and liners, and large surface warships are detected most often.

Since it often takes much time to determine the class of the object detected at the maximum distance, it is advisable to determine the CMP of the detected object for each class that is typical of the navigation area, and then choose the CMP corresponding to the actual class of the object, resulting from its classification procedure.

With this approach, the PDF of the detected object's velocity can be written in the form:

$$g_{V/\omega}\left(v\right)= norm\left(v \,||\, mV_{\omega}; sV_{\omega}\right), \quad (6)$$

where $norm(v||mV_{\omega}; sV_{\omega})$ is normal PDF of a random value V with mathematical expectation mV_{ω} and RMS deviation sV_{ω} (v is a non-random argument of the PDF).

It should be noted that the object's velocity can be clarified by isolating the sequence of discrete components of the shaft and blades sounds in the object's envelope spectrum and determining the propeller shaft rotation speed using these data [15]; a reference book with the stroke speed values depending on the propeller shaft(s) rotation speed for each class of the objects will also be useful in this case.

Heading of the object can be calculated by the formula [16]:

$$K = K_{\rho} - \arcsin\left[\frac{V_{н}}{V}\cdot\sin\left(K_{\rho}-K_{н}\right)\right], \quad (7)$$

where $V_н$, $K_н$ are the speed and heading of the observer; K_ρ is the object's relative heading calculated by the formula derived in Annex 1:

$$K_{\rho} = arctg\frac{\sin\left(P_k-P_1\right)\cdot\left(t_N-t_k\right)\cdot\sin\left(P_N\right)-\sin\left(P_N-P_k\right)\cdot\left(t_k-t_1\right)\cdot\sin\left(P_1\right)}{\sin\left(P_k-P_1\right)\cdot\left(t_N-t_k\right)\cdot\cos\left(P_N\right)-\sin\left(P_N-P_k\right)\cdot\left(t_k-t_1\right)\cdot\cos\left(P_1\right)}. \quad (8)$$

Here, P_1, P_k, P_N are the object's bearings from the array of bearings $P_1, \ldots, P_N$ measured on one tack of the object's straight-line motion at the time points $t_1, \ldots, t_N$; in order to improve the accuracy of the object's heading estimation, bearing P_k corresponding to time t_k should be approximately equal to the arithmetic mean of bearings P_1 and P_N.

Thus, the object's heading estimation is the function of its velocity and the array of bearings. Conditional (depending on the velocity) PDF of the object's heading has the form:

$$g_{\hat{K}/V}\left(k\right)= g_{\Delta K_{\rho}/V}\left(k - K_{\rho} + arcsin\left[\frac{V_{н}}{V}\cdot sin\left(K_{\rho}-K_{н}\right)\right]\right), \quad (9)$$

where $g_{\Delta K_\rho/V}(k)$ is a conditional PDF of the relative heading error calculated according (8) with RMS 3–5 deg.

It follows from (7) that, if the observer's velocity is much lower than that of the object, then the relative heading of the object is close to its heading, and is equal to it if the observer is not moving.

We will determine the distance to the target using both the energy (based on the absolute level of the object's signal) and the spectral (based on the tilt of the object's signal spectrum) methods [11]. These methods are implemented by measuring the object's signal levels in M frequency bands (usually three ones). Respective stochastic model is as follows:

$$\hat{W}_k = W_k(P_0, r) + \Delta W_k,\ k=1,...,M, \quad (10)$$

where $\hat{W}_k$ is the estimate of the object's signal level in the k-th frequency band (dB);

M is the number of frequency bands;

$W_k(P_0, r)$ is the non-random function relating the actual value of the object's signal level in the k-th frequency band (dB) and the object's standard noise level P_0 (dB), and the distance r to it (km);

ΔW_k is the relative error of measurement of the object's signal level in the k-th frequency band, dB. We assume that it is distributed according to the normal law with zero mathematical expectation and standard deviation $\sigma_{\Delta Wk}$ calculated by the formula derived in Annex 2:

$$\sigma_{\Delta W_k} = 10\cdot\lg\left(1 + \frac{\sqrt{(q_k+1)^2+1}}{q_k\cdot\sqrt{\Delta f_k\cdot T_k}}\right). \quad (11)$$

Here, q_k is SNR (according to power) at the output of the linear part of the receiving channel in the k-th frequency band;

Δf_k, T_k are the bandwidth, Hz, and the time of accumulation, sec, in the k-th frequency band, respectively.

Note that the denominator in the right part of (11) is equal to the indicator (output) SNR which is equal to 5 on average at the detection threshold. Therefore, the relative error of measurement of the threshold signal level $\sigma_{\Delta Wk}$ does not exceed 1.1 dB.

Function $W_k(P_0, r)$ can be represented as

$$W_k(P_0, r) = P_0 + K_k(r) + H_k, \quad (12)$$

where $K_k(r)$ is the transfer characteristic of the acoustic channel of the object's noise propagation from the emission point to the input of the receiving antenna in the k-th frequency band (dB), which is calculated by the approximated formula [17]:

$$K_k(r) \approx -20\cdot\lg f_{ekv_k} + 10\cdot\lg\left(f_{в_k} - f_{н_k}\right) - \beta\left(f_{ekv_k}\right)\cdot r + 10\cdot\lg A\left(f_{ekv_k}, r\right) - 20\cdot\lg r - 60, \quad (13)$$

$f_{н_k}, f_{в_k}$ are the lower and upper bound frequencies of the k-th frequency band;

$f_{ekv_k} = \sqrt{f_{н_k}\cdot f_{в_k}}$ is the equivalent frequency of the k-th frequency band;

$\beta(f)$ is the frequency dependence of the spatial attenuation coefficient for the sea area considered;

$A(f, r)$ is the frequency dependence of the signal propagation anomaly in the function of distance, expected depth of the object and the depth of the observer for the sea area considered;

H_k is the transmission coefficient of the receiving path of the sonar system in the k-th frequency band (dB). Due to the high accuracy of its measurement during the receiving path calibration, we will hereinafter consider this parameter to be a non-random value.

Since the only random value in the right part of (10) is the relative error ΔW_k of measurement of the object's signal level in the k-th frequency band, the conditional (depending on the object's standard noise level P_0 and the transmission coefficient $K_k(r)$) PDF of estimate of the signal level in the k-th frequency band can be written as follows [14]:

$$g_{\hat{W}_k/P_0,K(r)}(w_k) = g_{\Delta W_k}\left(w_k - P_0 - K_k(r) - H_k\right), \quad (14)$$

where $g_{\Delta Wk}(w_k)$ is the PDF of the signal level measurement error in the k-th frequency band.

Since the object's noise levels are measured in different frequency bands, their measurement errors are independent. As a result, the joint conditional PDF of the signal level estimates in M frequency bands is equal to the product of PDF of level estimates in individual frequency bands:

$$g_{\hat{W}_1,...,\hat{W}_M/P_0,K_1(r),...,K_M(r)}(w_1,...,w_M) = \prod_{k=1}^{M} g_{\Delta W_k}\left(w_k - P_0 - K_k(r) - H_k\right). \quad (15)$$

To determine the distance according to the maximum likelihood criterion, i.e., to obtain the maximum-likelihood estimate (ML-estimate) of the distance to the object, it is necessary to [18]:

– set the estimate of value $\hat{P}_0$ of the detected object and substitute it in (15) instead of the non-random parameter P_0;
– calculate the transmission coefficients $\hat{K}_k(r)$ versus the distance for each frequency band, and substitute them in (15) instead of the non-random dependencies $K_k(r)$;
– substitute the estimates of the object's noise level in the frequency band $\hat{W}_k$ in (15) instead of the non-random arguments w_k.

After these procedures, formula (15) will be transformed into the likelihood function which only depends on the distance to the target [18]:

$$g_{\hat{W}_1,\ldots,\hat{W}_M/r}\left(\hat{W}_1,\ldots,\hat{W}_M\right) = \\ = \prod_{k=1}^{M} g_{\Delta W_k}\left(\hat{W}_k - \hat{P}_0 - \hat{K}_k(r) - H_k\right). \quad (16)$$

The distance to the object, corresponding to the global maximum of the right part of (16) is the ML-estimate of the distance to the object:

$$\hat{R}_{opt} = \\ = \arg \max_{r} \prod_{k=1}^{M} g_{\Delta W_k}\left(\hat{W}_k - \hat{P}_0 - \hat{K}_k(r) - H_k\right). \quad (17)$$

In view of the fact that the PDF of the noise levels measurement errors in frequency band can be assumed normal PDF with zero mathematical expectations and standard deviations calculated by (11), formula (17) can be simplified by using the sum instead of the product and minimization instead of maximization in the right part:

$$\hat{R}_{opt} = \arg \min_{r} \sum_{k=1}^{M} \frac{\left(\hat{W}_k - \hat{P}_0 - \hat{K}_k(r) - H_k\right)^2}{\sigma^2_{\Delta W_k}}. \quad (18)$$

At the maximum distance, the object's noise, as a rule, cannot be detected in all frequency bands (in most cases it is detected in one band only), and this fact should be taken into account in (18) by introducing the indicators I_k of the object's noise detection in each frequency band; these indicators are equal to one if the object's noise is detected, and to zero if not:

$$\hat{R}_{opt} = \arg \min_{r} \sum_{k=1}^{M} I_k \cdot \frac{\left(\hat{W}_k - \hat{P}_0 - \hat{K}_k(r) - H_k\right)^2}{\sigma^2_{\Delta W_k}}. \quad (19)$$

Let us estimate the accuracy of determining the distance to the object using the algorithm (19). It is affected by three factors:

1. the accuracy of estimation of the detected object's standard noise level $\hat{P}_0$;
2. the accuracy of the transfer characteristic $\hat{K}_k(r)$ calculation for the acoustic channel of signal propagation in each frequency band;
3. the accuracy of signal level $\hat{W}_k$ measurement in each frequency band.

Let us consider these factors in detail.

The noise of detected object can be estimated accurately enough if the class of the detected object and the mean noise level of this class objects in the given navigation area are known. Therefore, it is advisable to act in the same way as with the velocity: to determine the distance to the object for each class of objects that can be met in the navigation area, by substituting the most probable noise level of respective class in the algorithm (19). It is easy enough to determine the distribution of noise level of each class of objects in each navigation area, using reference literature or experimental data analyses (e.g., [19]). For most classes of objects, the distribution of noise level can be approximated by normal PDF with some average value and a standard deviation of 2.0–2.5 dB.

The transfer characteristic of the acoustic channel of signal propagation in each frequency band $\hat{K}_k(r)$ is calculated by formula (13), where the frequency dependence of the spatial attenuation coefficient can be a source of errors if it had not been measured accurately enough for the navigation area. Another potential source of errors here is signal propagation anomaly which is calculated according to a ray program available in every modern submarine sonar system [20]. Studies have shown that the main source of errors in the anomaly calculation are inaccurate data on the acoustic conditions in the area (mainly the vertical sound speed distribution), as well as the depth of the object if it is a submarine. The studies have also shown that the error in the calculation of transfer characteristic of the signal propagation channel does not fluctuate along the distance and behaves like a constant error of displacement. If the submarine is equipped with the sound speed meters with break-wire sensors that can measure the vertical distribution of the sound speed down to a depth of 1000 m with high accuracy [21], the error in the transfer characteristic calculation for the propagation channel can be approximated well with a normal distribution law with zero mathematical expectation and a standard deviation of 1.5 dB for a surface object and 2.0 dB for a submarine.

The distribution of the signal level measurement error also obeys the presumably normal law with zero mathematical expectation and the standard deviation determined by formula (11). It should be noted here that this error makes the smallest contribution in the error in determining the distance to the object. As a result, increasing accuracy of signal levels measurement in the frequency band with increasing SNR will not practically affect the accuracy of determining the distance, because the two considered factors do not depend on time and SNR.

Therefore, it should be indicated in (19) that the expected standard noise level of the object and the transfer characteristic of noise propagation, as well as the distance estimate itself depend on the object's class ω:

$$\hat{\mathrm{R}}_{\mathrm{opt}/\omega} = = \arg \min_{\mathrm{r}} \sum_{k=1}^{M} I_k \cdot \frac{\left(\hat{W}_k - \hat{P}_{0/\omega} - \hat{K}_{k/\omega}(r) - H_k\right)^2}{\sigma^2_{\Delta W_k}}. \quad (20)$$

Since the distance likelihood function can be considered as the PDF of distance under the condition of normalization, it can be used for estimating the a posteriori RMS error of the estimated distance to the object [10, 18]:

$$\sigma^2_{\hat{\mathrm{R}}_{\mathrm{opt}/\omega}} = \frac{\int\limits_{mP_{0/\omega}-2sP_{0/\omega}}^{mP_{0/\omega}+2sP_{0/\omega}} g_{P_0}(p) \cdot \int\limits_{-2\sigma_{\Delta K_\omega}}^{2\sigma_{\Delta K_\omega}} g_{\Delta K}(\Delta k) \cdot \left[\mathrm{R}(p, \Delta k) - \hat{\mathrm{R}}_{\mathrm{opt}/\omega}\right]^2 \cdot dp \cdot d\Delta k}{\int\limits_{mP_{0/\omega}-2sP_{0/\omega}}^{mP_{0/\omega}+2sP_{0/\omega}} g_{P_0}(p) \cdot \int\limits_{-2\sigma_{\Delta K_\omega}}^{2\sigma_{\Delta K_\omega}} g_{\Delta K}(\Delta k) \cdot dp \cdot d\Delta k}, \quad (21)$$

where $R(p, \Delta k)$ is the estimate of distance to the object, calculated by (20) with p substituted instead of $\hat{P}_0$, and $\hat{K}_k(r) + \Delta K$ instead of $\hat{K}_k(r)$.

As a result, the PDF of the estimate of distance to the object at time point t_j can be represented as follows:

$$g_{R_j/\omega}(r) = norm\left(r \,\|\, R_{opt/\omega}; \sigma_{R_{opt/\omega}}\right). \quad (22)$$

Now all the data have been obtained for constructing the conditional (depending on the velocity, heading and distance) joint PDF of bearings estimates:

$$\begin{aligned} & g_{\hat{P}_1,\ldots,\hat{P}_N,K,V,R_j}(p_1,\ldots,p_N,k,v,r) = \\ & = g_{R_j/\omega}(r) \cdot g_{V/\omega}(v) \cdot g_{\hat{K}/V}(k) \cdot g_{\hat{P}_1,\ldots,\hat{P}_N/K,V,R_j}(p_1,\ldots,p_N) = \\ & = norm\left(r \,\|\, R_{opt/\omega}; \sigma_{R_{opt/\omega}}\right) \cdot norm\left(v \,\|\, mV_\omega; sV_\omega\right) \times \\ & \times g_{\Delta K_\rho/V}\left(k - K_\rho + \arcsin\left[\frac{V_н}{v} \cdot \sin\left(K_\rho - K_н\right)\right]\right) \cdot \prod_{i=1}^{N} norm\left(p_i - P\left(t_i, k, v, r_j\right) \,\|\, 0; \sigma_{P_i}\right). \end{aligned} \quad (23)$$

Heading, velocity and distance corresponding to the global maximum of the PDF (23) are the most likely estimates of the CMP of the class ω object:

$$\begin{aligned} & \left(\hat{K}_{opt}, \hat{V}_{opt}, \hat{R}_{opt/j}\right) = \arg \max_{k,v,r} g_{\hat{P}_1,\ldots,\hat{P}_N,K,V,R_j}(p_1,\ldots,p_N,k,v,r) = \\ & = \arg \max_{k,v,r} \left\{ norm\left(r \,\|\, R_{opt/\omega}; \sigma_{R_{opt/\omega}}\right) \cdot g_{\Delta K_\rho/V}\left(k - K_\rho + \arcsin\left[\frac{V_н}{v} \cdot \sin\left(K_\rho - K_н\right)\right]\right) \times \right. \\ & \left. \times norm\left(v \,\|\, mV_\omega; sV_\omega\right) \cdot \prod_{i=1}^{N} norm\left(p_i - P\left(t_i, k, v, r_j\right) \,\|\, 0; \sigma_{P_i}\right) \right\}. \end{aligned} \quad (24)$$

Due to the fact that all PDF in the right part of (24) are normal, formula (24) can be simplified by replacing the products with sums and the maximization with minimization:

$$\left(\hat{K}_{opt},\hat{V}_{opt},\hat{R}_{opt/j}\right)=$$

$$=arg\ \min_{k,v,r}\left\{\frac{\left(r-R_{opt/\omega}\right)^2}{\sigma^2_{R_{opt/\omega}}}+\frac{\left(v-mV_\omega\right)^2}{sV^2_\omega}+\frac{\left(k-mK\right)^2}{sK^2_\rho}+\sum_{i=1}^{N}\frac{\left(p_i-P\left(t_i,k,v,r_j\right)\right)^2}{\sigma^2_{P_i}}\right\} \quad (25)$$

where $mK=K_\rho-\arcsin\left[\frac{V_и}{v}\cdot\sin\left(K_\rho-K_и\right)\right]$.

Since there is no reason to consider the minimized functional as single-modal, the minimum should be found by simple search of the heading, velocity and distance values.

The idea of algorithm (25) is that the array of bearing estimates updates the values of heading, velocity and distance estimates by adding them in the motion model (1) and (3). However, this does not happen if the bearing in the problem solution interval does not change or changes very slowly compared to its measurement error. The reasons for that may be a large distance to the object or the object's close approach heading. In these cases, the object's CMP estimates can be as follows:

- average velocity mV_ω of the object of class ω can be taken as the velocity estimate;
- reverse bearing of the object, equal to the relative heading in case of close approach heading can be taken as the relative heading estimate;
- the heading estimate is calculated by formula (7) using the velocity and relative heading estimates;
- the distance calculated by algorithm (20) can be taken as the distance estimate.

As was mentioned above, the problem of determining the CMP is solved cyclically with the period of bearings measurement. At the end of each cycle of the problem solution, the object's velocity, distance and heading estimates obtained in L previous cycles (including the current one) are smoothed using a robust algorithm which consists in the following:

- all L estimates of the same parameter are ranked in the ascending or descending order of their values;
- 10-20% of the minimum and maximum estimates are removed from the ranked series;
- the remaining estimates are used for calculating the arithmetic mean which is attributed to the time corresponding to mean time L of the initial estimates.

To further reduce the fluctuations of the CMP estimates, the smoothing procedure can be repeated twice.

In each N-th cycle of the problem solution, the presence of the observed object's maneuver is checked. The decision about the maneuver is made if at least one of the following events is detected [22];

1. the bearing change rate has changed sharply, which means that the object is maneuvering to change its heading or velocity. Relevant decision can be made if the following condition is fulfilled:

$$\left|\hat{P}_{N-S}+\dot{P}_{L-S}\cdot\left(t_N-t_{N-S}\right)-\hat{P}_N\right|>4.5\cdot\sigma_P, \quad (26)$$

where $\dot{P}_{L-S}$ is the bearing change rate calculated by the formula

$$\dot{P}_{L-S}=\frac{\hat{P}_{N-S}-\hat{P}_{N-L}}{t_{N-S}-t_{N-L}}, \quad (27)$$

N is the number of the last measurement of the bearing;

σ_P is RMS error of the bearing estimate,

$S=1\ldots3$; $L=20\ldots30$;

2. the speed of propeller shaft(s) rotation has changed according to the monitoring of the sound of discrete components in the object's noise envelope spectrum, due to which the object's velocity has changed. Relevant decision can be made if the following condition is fulfilled in two sequential cycles of calculation of the object's noise envelope spectrum:

$$\left|\tilde{F}-F_N\right|>3\cdot\sigma_F, \quad (28)$$

where $\tilde{F}$ is the average speed of the object's propeller shaft rotation measured in previous B cycles of calculation of the object's noise envelope spectrum, except for two last cycles;

F_N is the speed of the object's propeller shaft rotation measured in the last cycle of calculation of the object's noise amplitude envelope;

σ_F is the RMS error of propeller shaft rotation frequency measurement, equal to the resolution of the spectral analysis of the object's noise amplitude envelope;

3. sound sequences of discrete components with close frequency have suddenly appeared in the envelope spectrum, which means that the object is performing the heading change maneuver. This event is detected during the analysis of the object's noise amplitude envelope spectrum;
4. the object's noise level has changed sharply, which means that the object's velocity or depth have changed. Relevant decision can be made if the following condition is fulfilled:

$$\left|\tilde{W}_N - \hat{W}_N\right| > 4.5 \cdot \sigma_{\Delta W}\,, \qquad (29)$$

where $\tilde{W}_N$ is the average level of the object's noise, calculated in the highest frequency band in previous F cycles of measuring the signal levels in the frequency bands, except for three last cycles;

$\hat{W}_N$ is the object's noise level measured in the same frequency band in the last cycle;

$\sigma_{\Delta W}$ is the RMS error of the object's noise level measurement in the same frequency band.

3. ALGORITHM DESCRIPTION

To ensure the algorithm operation, the following data should be recorded in the sonar system memory for each class ω of marine objects that can be potentially detected in the given area:

- average velocity $m_{V/\omega}$ and its standard deviation $\sigma_{V/\omega}$;
- dependence $V_\omega(Y_{PS})$ of motion velocity V_ω on the speed of propeller shaft(s) rotation Y_{PS};
- dependence of the mean value of standard noise level on the motion velocity $mP_{0/\omega}(V)$ and its standard deviation $sP_{0/\omega}(V)$.

Upon arrival at the area, using a ray program for the acoustic conditions in the area, the transfer characteristics of the object's noise propagation channel $K_{k/\omega}(r)$, $k = 1, \ldots, M$ in each frequency band are calculated for each class of objects and the current depth of the observer submarine.

The CMP of detected object are determined simultaneously for each class of objects that can be potentially detected in the navigation area.

When solving this problem with regard to a particular class, the noise level and velocity specifications of the object are selected from the tables and graphs prepared for the given navigation area; calculated transfer characteristics of the object's noise propagation channel in each frequency band are used as well. Average velocity of an object of the considered class $V_\omega = m_{V/\omega}$ was taken as the object's velocity.

The problem solution starts from the moment of the object detection with the cycle of input data processing in passive mode (t_1 is the time point of the object detection, t_N is the time point of the current N-th cycle). The following procedures are implemented in each cycle:

1. it is determined from (26)–(29) whether the object is maneuvering or not. If a maneuver is detected, this is considered as the start of a new straight-line tack of the object. The last CMP estimates are taken as the initial approximations of the CMP on the current tack;
2. the object's bearing and SNR in each frequency band are measured, as well as the noise levels in the frequency bands where the SNR exceeds the threshold value;
3. RMS error of bearing estimate σ_P is calculated by formula (2), taking into account the measured SNR;
4. RMS errors of the object's noise level estimates $\sigma_{\Delta Wk}$ are calculated by formula (11) for each frequency band;
5. if the condition

$$\left|P_N - P_1\right| > 4.5 \cdot \sigma_P \qquad (30)$$

is fulfilled, i.e., when the magnitude of bearing change from the moment of the object detection till the current time point t_N has exceeded the maximum error of the bearings difference, the estimate of the object's relative heading is found by formula (8). Before this point, reverse bearing is taken as the object's relative heading. The reason is that there is no statistically verified change of bearing within the observation time interval either because of a too large distance to the object (in which case the accuracy of determining the object's relative heading is not very important), or because the object is moving with close approach heading (in which case the object's relative heading is equal to its reverse bearing);

6. taking into account the target's velocity $V=mV_\omega$ and the observer's motion parameters, the estimate of the object's heading is calculated by formula (7);
7. the estimate of distance to the object is determined by formula (20) and its RMS error is calculated by formula (21);
8. the estimates of bearings and average values of velocity, heading and distance are substituted in (25), and the global minimum of this function is calculated. The global minimum is searched for within the range of possible velocity, heading and distance values. As was mentioned above, this is done by direct search of the velocity, heading and distance values in the indicated intervals. It has been found from simulation that each interval should be divided into at least 20 values. After the global minimum of the likelihood function (25) has been found, the corresponding values of velocity, heading and distance are taken as their most likely estimates in the current cycle of the problem solution;
9. the estimates of the object's velocity, distance and heading, obtained in the last L cycles of the problem solution are smoothed with the robust algorithm described above.

The algorithm output at each cycle of the problem solution is the estimates of the observed object's CMP, corresponding to each class of objects that may be potentially detected in the navigation area. This allows each decision on the observed object class to be supported with the CMP estimates corresponding to this class. It should be noted that the multi-class strategy of determining the CMP should be applied within the whole period of the object observation, because there is a probability that the decision of the object's class will change during its observation.

4. ALGORITHM SIMULATION

The algorithm was simulated in order to assess the accuracy it provides in determining the detected object's CMP.

Simulation was carried out by repeating the same situation when the sonar system carrier was approached by a simulated object with known noise and velocity characteristics in specified acoustic conditions. Each repetition consisted of the object detection, which was fixed if the noise level exceeded the threshold value in at least one frequency band of the sonar system receiver, and cyclic (with specified period) determination of the object's CMP. At each repetition, in accordance with the PDF depending on the object class, the detected object's velocity and noise were modeled at random, as well as the error of the channel transfer characteristic calculation; at the same time, at each cycle of the CMP determination, the measurement errors of bearing and noise levels in each frequency band of the receiver were modeled. Following the strategy described above, the CMP of the detected object were determined simultaneously for several possible classes of the object.

After each repetition, time dependencies of the estimates of the object's velocity, heading and the distance to the object, as well as their errors, were formed for each possible class of the object. The results of all events were used for calculating the time dependencies of RMS errors of heading, velocity and distance, taking into account the influence of all factors.

Simulation was carried out for the following conditions.

1. The navigation area was an ocean area located far from the coast, where, with some minor exceptions, only large commercial vessels (mainly transport vessels), large warships of cruiser and frigate classes, and nuclear-powered submarines can be detected. The mean values (mV_ω, $mP_{0/\omega}$) and RMS ranges (σV_ω, $\sigma P_{0/\omega}$) of their velocities and noise are presented in Table 1 ($mP_{0/\omega}$ is the noise pressure measured in dB//re 20 µPa in the 1 Hz band at the frequency 1 kHz and 1 m from the source). Figure 1 shows corresponding averaged dependencies of reduced noise on the motion velocity.

Table 1. Noise and velocity characteristics of three classes of objects

ω	**Class**	mV_ω**, m/s**	σV_ω**, m/s**	$mP_{0/\omega}$**, dB**	$\sigma P_{0/\omega}$**, dB**
1	Nuclear-powered submarine (NPS)	3.75	0.63	62.0	2.5
2	Large surface warship (SW)	9.0	1.0	100.0	2.5
3	Large transport vessel (TV)	7.5	1.0	107.0	2.5

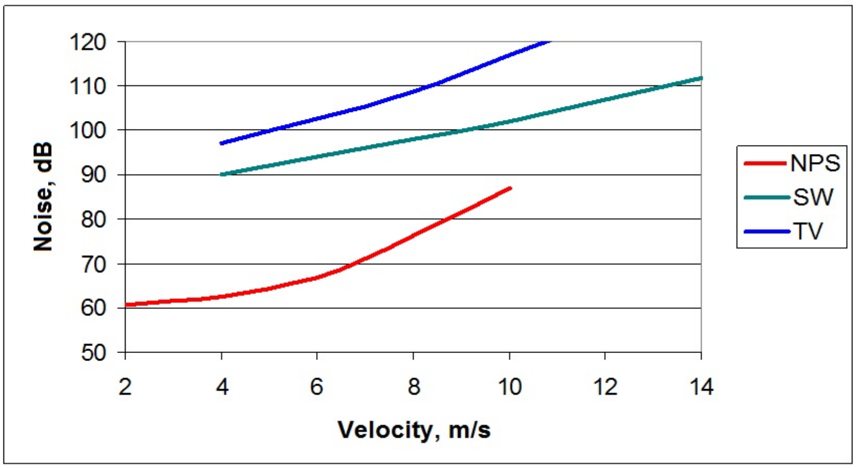


Fig. 1. Averaged dependencies of reduced noise of marine objects on their motion velocity.

2. A nuclear-powered submarine and a transport vessel were taken as detected objects for which the CMP were determined.
3. In the course of solution, the target submarine was moving at the depth of 200 m along two straight-line tacks at the speed of 4 m/s. The heading was 270 deg on the first tack, and 210 deg on the second one. Actual noise of the submarine at 4 m/s was 63 dB. Each tack lasted for 20 minutes.
4. The submarine was detected at the maximum distance of 11.2 km, with the observer submarine's relative bearing being 26 deg to port.
5. During observation, the transport vessel was also moving along two straight-line tacks lasting 20 minutes each, with headings 300 and 260 deg. The velocity on the first tack was 6.5 m/s and the noise was 105 dB. On the second tack, the velocity increased to 8.5 m/s and the noise—to 109 dB.
6. Two scenarios of the transport vessel detection were considered. In the first scenario, the transport vessel was detected at the maximum distance of detection (110 km), and in the second one—at the distance of 30 km when leaving the line with other surface ships. In both scenarios, detection was carried out at the relative bearing to port of the observer submarine.
7. In all episodes, the observer submarine moved rectilinearly and uniformly at the depth of 100 m, at the speed of 2.5 m/s, and with 90 deg heading.
8. Vertical distribution of the sound speed in the area is shown in Fig. 2. The reduced level of the sea noises is 31 dB.

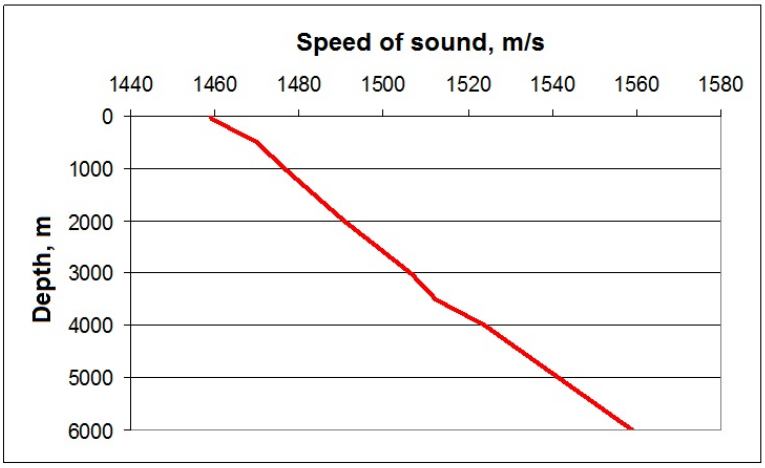


Fig. 2. Vertical distribution of the sound speed in the area.

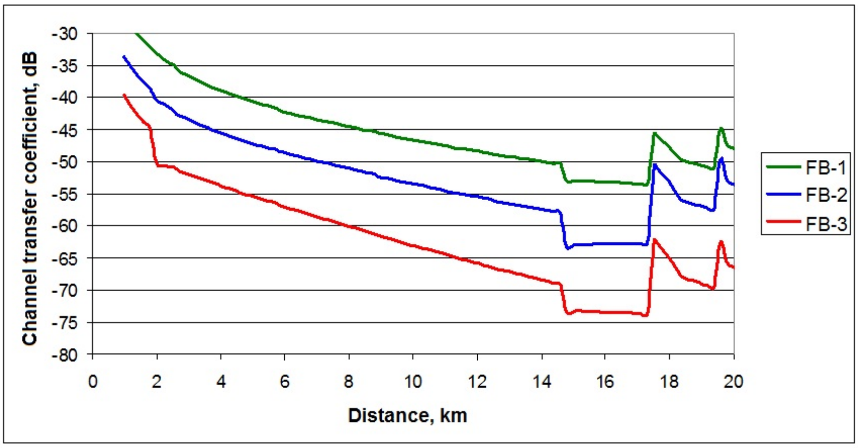
The initial data for each episode simulation were the following:

- the actual class of the observed object;
- the number of tacks of the observed object's motion, and the duration of each tack;

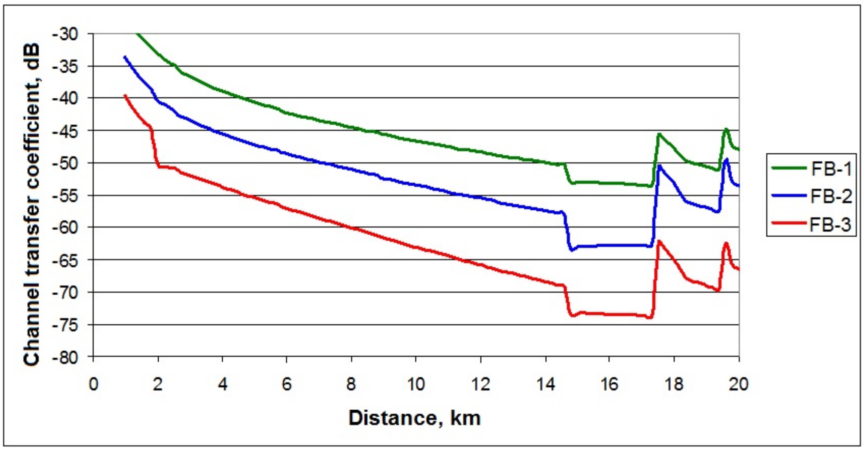
- the initial Cartesian coordinates, heading and depth of the observed object on each tack; the object's velocity and noise varied within the limits corresponding to the class of the simulated object;
- the initial Cartesian coordinates, heading, velocity, and depth of the observer submarine in the course of the problem solution.

The CMP of the detected object were determined in each episode by a multi-class strategy, i.e., simultaneously for three classes of objects that could be detected in the navigation area (Table 1).

Before starting the simulation using a beam-tracing program for the specified acoustic conditions, actual transfer characteristics of noise propagation channel in each of three frequency bands were calculated for each of the three classes of objects $K_{k/\omega}(r)$, $k = 1, \ldots, 3$, $\omega = 1, \ldots, 3$ (Fig. 3).



a) transfer characteristics for the class of submarines



b) transfer characteristics for the class of transport vessels

Fig. 3. Transfer characteristics of the object's noise propagation channel for the classes of submarines and transport vessels in each of three frequency bands

Simulation of each of the two classes consisted of 11 × 11 × 11 = 1331 repetitions (experiments) which differed in the values of actual velocity V and noise P_0 of the object, as well as the error ΔK_ω in calculating the transfer characteristic of the signal propagation channel; all these values were searched in the form of three nested cycles within their variation intervals.

The error ΔK_ω in calculating the transfer characteristic of the signal propagation channel was added to the calculated actual transfer characteristics $K_{k/\omega}(r)$:

$$\hat{K}_{k/\omega}(r) = K_{k/\omega}(r) + \Delta K_\omega . \quad (31)$$

The CMP were estimated in each experiment with a 10 seconds' cycle. Each cycle included the following procedures.

1. Using the well-known formulas of joint maneuvering of marine objects [1, 16], actual values of current Cartesian coordinates of the observed object and the observer submarine, distances between them, and the object's bearings were calculated.

2. Using the actual dependencies of the transfer characteristics of the object's noise propagation channel in the frequency bands $K_{k/\omega}(r)$, $k = 1, \ldots, M$ on the

distances, the actual coefficients of transmission, corresponding to the current distance to the simulated object were determined.

3. Using the actual noise of the object and the actual transmission coefficients of the channel, the actual values of the object's noise levels in each of three frequency bands were calculated by formulas (12) and (13) (black lines in Fig. 4), and SNR in the same frequency bands were also calculated, taking into account the sea noise level (Fig. 5). Further, in this cycle of simulation, only the frequency bands where the calculated SNR exceeded the detection threshold were taken into consideration.

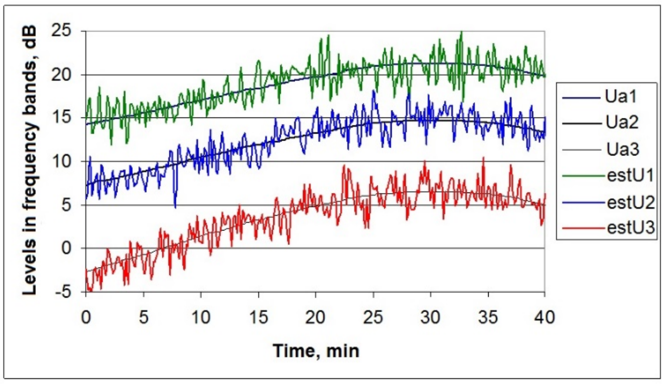


Fig. 4. Dependencies of actual values of the object's noise levels in three frequency bands (black solid lines) and their estimates (colored lines) on the time of submarine observation.

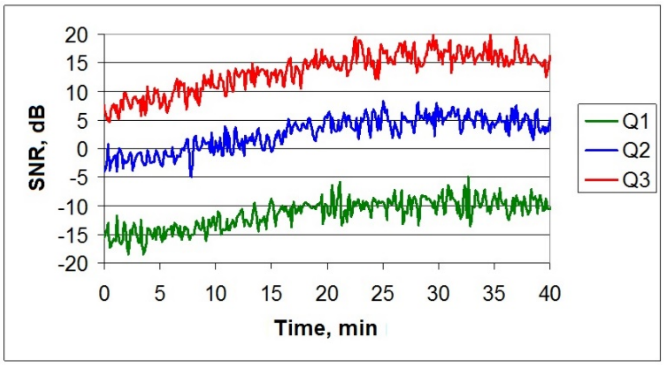


Fig. 5. Dependencies of SNR estimates in three frequency bands on the time of submarine observation.

4. Based on to the SNR values, RMS errors of the object's noise levels measurement σΔWk in the frequency bands where the SNR exceeded the threshold value were calculated by formula (11). The resulting RMS error values were used for simulating the errors in the signal level measurements in the frequency bands, which were superimposed on the true coefficients of transmission (colored lines in Fig. 4).

5. According to algorithm (20), the estimate of distance to the object $\hat{R}_{opt/\omega}$ was determined (Fig. 6), and its RMS error $\sigma_{\hat{R}_{opt/\omega}}$ was calculated by formula (21).

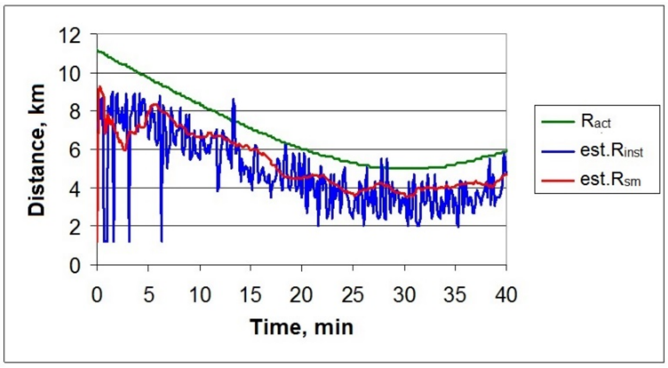


Fig. 6. Dependencies of the actual distance to the object (green line) and its instant (blue line) and smoothed (red line) estimates on the time.

6. In accordance with the SNR value in the highest frequency band where the SNR exceeded the threshold value, the RMS error of the object's bearing measurement was calculated by formula (2). Based on the result, the object's bearing estimate $\hat{P}_i$ was simulated in this cycle of the problem solution (Fig. 7).

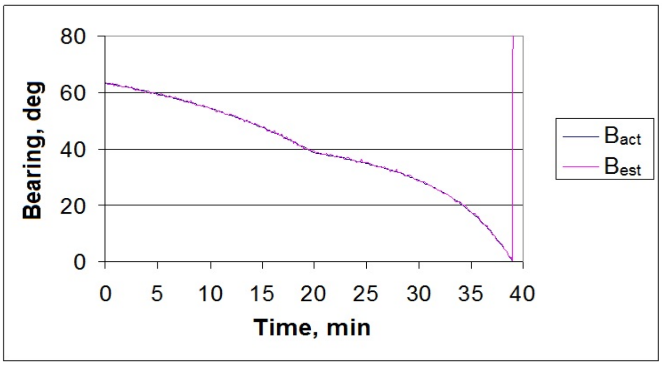


Fig. 7. Dependencies of the object's actual bearing and its estimate on the time.

7. Using the bearing estimates, the object's relative heading was calculated by formula (8) (Fig. 8), and its heading was calculated by formula (7) using the velocity estimate (Fig. 9).

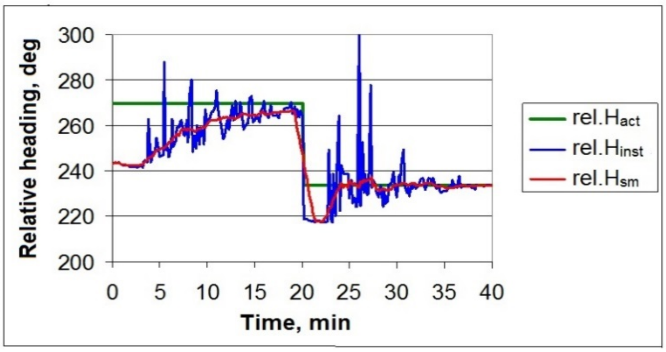


Fig. 8. Dependencies of the actual relative heading of submarine (green line), as well as its instant (blue line) and smoothed (red line) estimates on the time

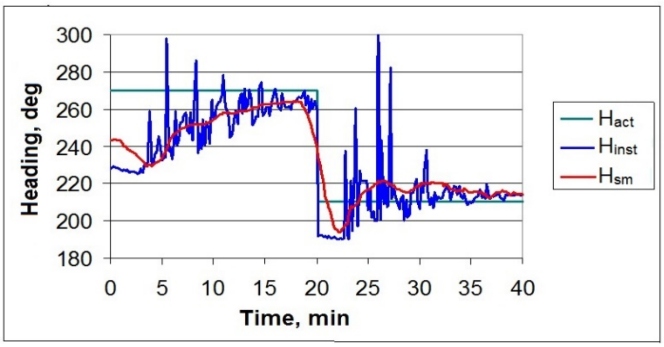


Fig. 9. Dependencies of the actual heading of submarine (green line), as well as its instant (blue line) and smoothed (red line) estimates on the time.

8. If the object's bearing changed by more than triple RMS error of its estimate, the estimates of heading, velocity and distance to the object were determined by algorithm (25) (Figs. 10, 11). Otherwise, the estimates mentioned in the discussion of formula (25) were taken as the object's CMP estimates.

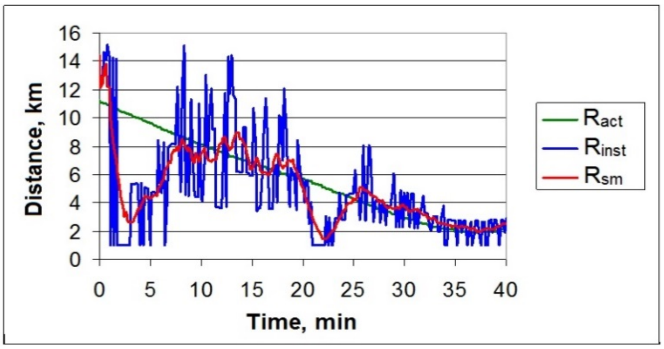


Fig. 10. Dependencies of the actual distance to submarine and its instant and smoothed estimates on the time.

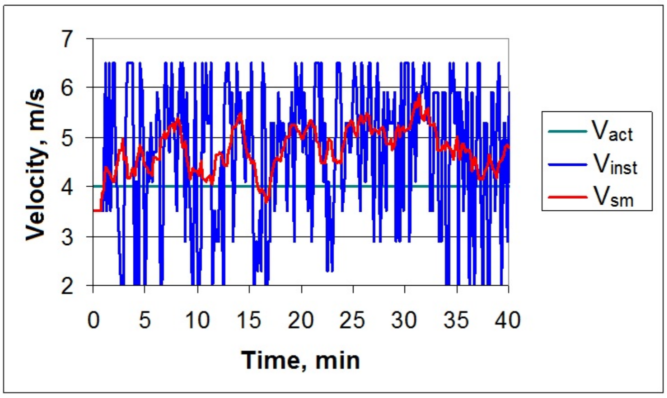


Fig. 11. Dependencies of the actual velocity of submarine and its instant and smoothed estimates on the time.

9. The estimates obtained in 19 neighboring cycles were smoothed with a robust algorithm which consisted in ranking all 19 estimates in the ascending order of their values, excluding two largest and two least estimates, averaging the remaining estimates, and attributing the result to the average of 19 cycles (Figs. 10, 11).

Using the dependencies of smoothed estimates of the object's heading, velocity and distance on the time from the moment of the object detection, resulting from 11^3=1331 experiments (each consisting of 11 simulation cycles of the velocity, noise level and error in calculating the channel transfer characteristic), the dependencies of RMS errors of the CMP smoothed estimates on the time were calculated:

$$\sigma_K^2(t_i) = \frac{\sum_{n=1}^{11}\left\{g_{V/\omega}(V_n)\cdot\sum_{m=1}^{11}g_{P_0/\omega}(P_{0/n})\cdot\left\{\sum_{s=1}^{11}g_{\Delta K/\omega}(\Delta K_s)\cdot\Delta\tilde{K}^2_{n,m,s}(t_i)\right\}\right\}}{\sum_{n=1}^{11}\left\{g_{V/\omega}(V_n)\cdot\sum_{m=1}^{11}g_{P_0/\omega}(P_{0/n})\cdot\left\{\sum_{s=1}^{11}g_{\Delta K/\omega}(\Delta K_s)\right\}\right\}}, \tag{32}$$

$$\sigma_V^2(t_i) = \frac{\sum_{n=1}^{11}\left\{g_{V/\omega}(V_n)\cdot\sum_{m=1}^{11}g_{P_0/\omega}(P_{0/n})\cdot\left\{\sum_{s=1}^{11}g_{\Delta K/\omega}(\Delta K_s)\cdot\Delta\tilde{V}^2_{n,m,s}(t_i)\right\}\right\}}{\sum_{n=1}^{11}\left\{g_{V/\omega}(V_n)\cdot\sum_{m=1}^{11}g_{P_0/\omega}(P_{0/n})\cdot\left\{\sum_{s=1}^{11}g_{\Delta K/\omega}(\Delta K_s)\right\}\right\}}, \tag{33}$$

$$\sigma_R^2(t_i) = \frac{\sum_{n=1}^{11}\left\{g_{V/\omega}(V_n)\cdot\sum_{m=1}^{11}g_{P_0/\omega}(P_{0/n})\cdot\left\{\sum_{s=1}^{11}g_{\Delta K/\omega}(\Delta K_s)\cdot\Delta\tilde{R}^2_{n,m,s}(t_i)\right\}\right\}}{\sum_{n=1}^{11}\left\{g_{V/\omega}(V_n)\cdot\sum_{m=1}^{11}g_{P_0/\omega}(P_{0/n})\cdot\left\{\sum_{s=1}^{11}g_{\Delta K/\omega}(\Delta K_s)\right\}\right\}}, \tag{34}$$

where $\sigma_K(ti)$, $\sigma_V(ti)$, $\sigma_R(ti)$ are the RMS errors of smoothed estimates of heading, velocity and distance, respectively, corresponding to the time point t_i counted from the moment of the object detection;

$g_V(V_n)$ is the value of PDF of the object's actual velocity V_n;

$g_{P_0}(P_{0/m})$ is the value of PDF of the object's noise $P_{0/m}$;

$g_{\Delta K}(\Delta K_s)$ is the value of PDF of error ΔK_s in calculating the transfer characteristic of the signal propagation channel;

$\Delta\tilde{V}_{n,m,s}(t_i), \Delta\tilde{K}_{n,m,s}(t_i), \Delta\tilde{R}_{n,m,s}(t_i)$ are the errors of smoothed estimates of velocity, heading and distance, respectively, corresponding to the time point t_i, obtained with the actual velocity V_n, actual noise $P_{0/m}$ and with the error in calculating the transfer characteristic of the signal propagation channel ΔK_s.

The results of calculations by formulas (32)–(34) are presented in Figs. 12–17.

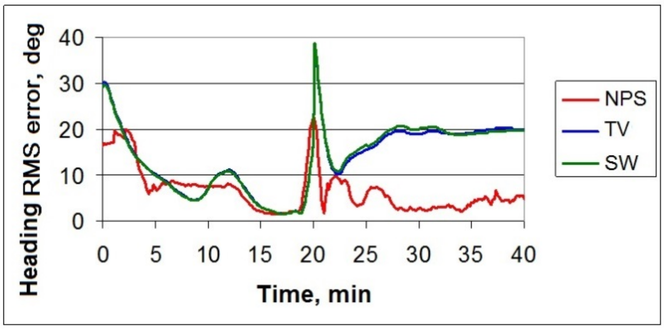


Fig. 12. Dependencies of RMS error of the object's heading on the time for three hypotheses about its class. The actual class of the object is submarine.

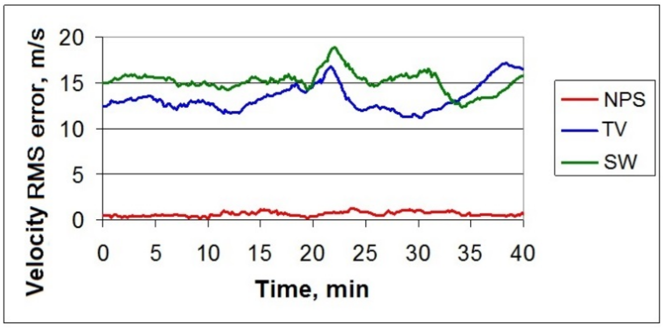


Fig. 13. Dependencies of RMS error of the object's velocity on the time for three hypotheses about its class. The actual class of the object is submarine.

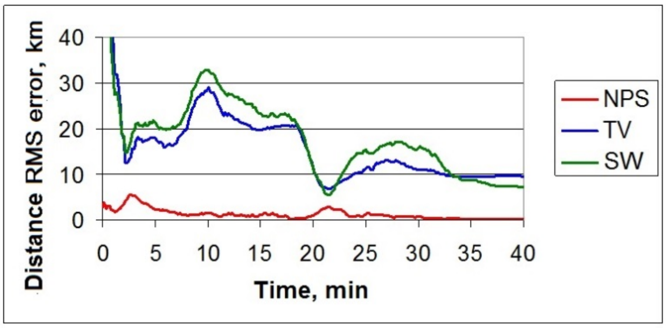


Fig. 14. Dependencies of RMS error of the distance to the object on the time for three hypotheses about its class. The actual class of the object is submarine.

Figures 12–14 show the RMS errors of smoothed estimates of the object's heading, velocity and distance, respectively, for three hypotheses about its class. The actual class of the object is submarine, which is confirmed by small values of errors corresponding to this class. It should be noted that sharp increase in the RMS error of the object's heading estimate is associated with the tack change at the 21st minute.

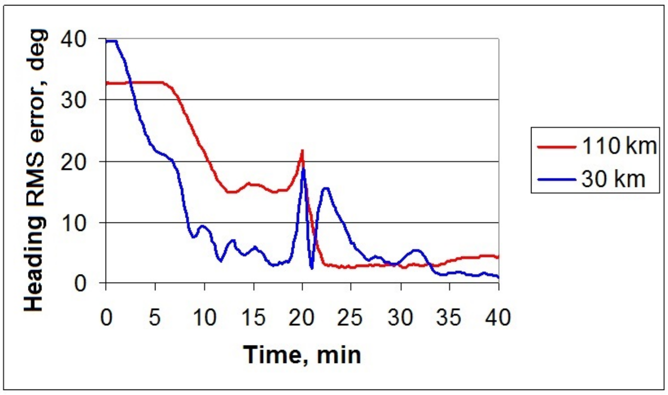


Fig. 15. Dependencies of RMS error of heading of a transport class object detected at the distances of 110 and 30 km.

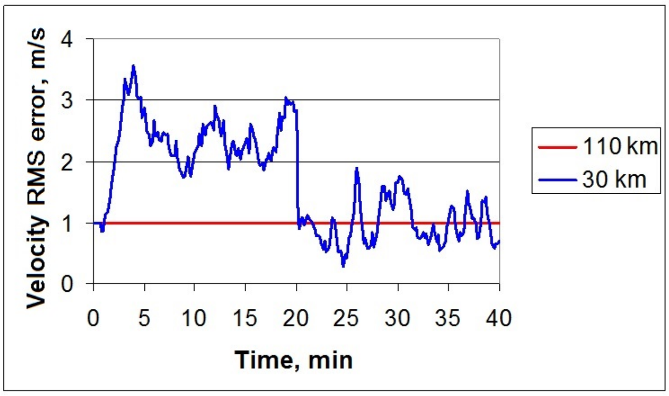


Fig. 16. Dependencies of RMS error of velocity of a transport class object detected at the distances of 110 and 30 km.

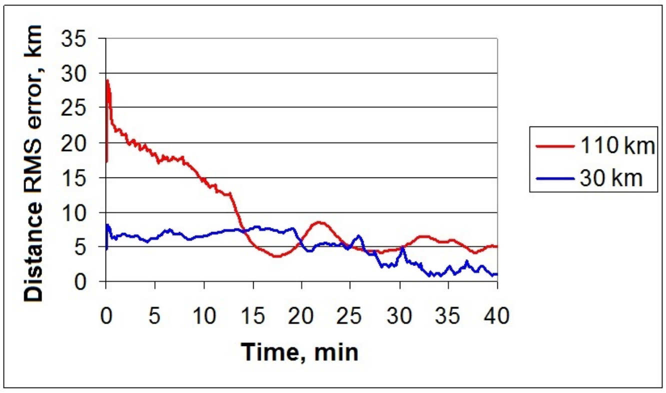


Fig. 17. Dependencies of RMS error of distance to a transport class object detected at the distances of 110 and 30 km.

Figures 15–17 show RMS errors of smoothed estimates of heading, velocity and distance, respectively, for an object of transport class, detected at the distances of 110 and 30 km. In the first case, detection took place at the maximum distance, while in the second case the transport vessel was detected when leaving the line with another vessel located closer to it. The fundamental difference between these cases consists in the rate of the object's bearing change. In the first case it was 0.05 deg/min (i.e., the bearing practically did not change over time), and in the second case it was 1 deg/min.

Having studied Figs. 12–17, the following conclusions can be drawn:

1. As expected, the minimum errors in determining the CMP of a detected object are observed when the hypothesis about its class coincides with the actual class of the object. This fact confirms the expediency of the proposed strategy, according to which the CMP of a detected object are determined simultaneously for several hypotheses about its class and then the CMP are selected corresponding to the class identified as a result of the detected object classification.
2. When the object is detected, the CMP errors reduce from the initial to steady-state values within the first 5–10 minutes.
3. When the object is detected at a distance of 30 km, i.e., when the bearing change rate is 0.5–1.0 deg/min or more, the average steady-state values of RMS errors of the CMP are:
    - heading 5–7 deg;
    - velocity 1.0–1.5 m/s;
    - distance 2–3 km.
4. When the object is detected at larger distances, when the bearing change rate does not exceed 0.1 deg/min, the CMP are determined by an optional algorithm that does not account for the bearing change behavior. In this case, the RMS errors in determining the CMP are within the following ranges:
    - heading 5–30 deg;
    - distance up to 20%;
    - RMS error of velocity is equal to the standard deviation of velocity within the corresponding class of objects.

CONCLUSIONS

1. An automatic algorithm has been proposed for determining the coordinates and motion parameters (CMP) of a marine object detected by a submarine sonar system in noise bearing finding mode, which does not require any special maneuvering. The algorithm is based on the idea that the problem can be solved using the estimates of velocity, heading and distance to the object, obtained by different methods, in addition to a traditionally used row of bearings. Since the accuracy of these estimates depends to a great extent on the class of the detected object and it may take much time to determine this class, it is proposed to first determine the CMP for all classes of objects that may be detected in the navigation area and then select the CMP estimates which match the class resulting from the object classification procedure.

2. The idea of involving the additional information such as the estimates of velocity, heading and distance to the object to determine the CMP makes it possible to obtain the result even when the bearing of the detected object remains almost unchanged.

3. Based on the simulation performed in this study, RMS errors in determining the CMP were estimated. According to the results, when an object is detected at a distance up to 30 km, i.e., when the bearing change rate is 0.5–1.0 deg/min or more, the steady-state average values of RMS errors in determining the CMP are as follows:

- heading 5–7 deg;
- velocity 1.0–1.5 m/s;
- distance 2–3 km.

When the object is detected at larger distances, the bearing change rate does not exceed 0.1 deg/min, and the problem is solved using an algorithm that does not deal with the bearing change dynamics, the RMS errors in determining the CMP lie within the following ranges:

- heading 5–30 deg;
- distance up to 20%;
- RMS error of velocity is equal to the standard deviation of objects' velocity within the respective class.

FUNDING

This work was supported by ongoing institutional funding. No additional grants to carry out or direct this particular research were obtained.

CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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Annex 1

FORMULA DERIVATION FOR CALCULATING THE OBJECT'S RELATIVE HEADING BY BEARINGS

The relative heading of an object moving rectilinearly and uniformly can be determined by three or more bearings measured in some arbitrary (not necessarily equal) time intervals.

Let us start with the derivation of formula for three bearings.

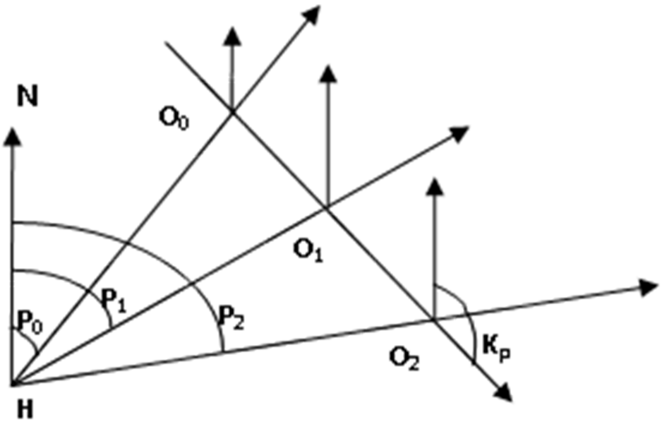

Fig. A1.1. Illustration explaining the formula derivation for determining the object's relative heading by three bearings: H is the observer's location; O_i is the object's location at time point t_i; N is direction to the North; P_i is the object's bearing at time point t_i, K_ρ is the object's relative heading.

According to the law of sines, from the triangles HO_0O_1 and HO_0O_2 we obtain:

$$\frac{\sin(P_1-P_0)}{V_\rho\cdot(t_1-t_0)}=\frac{\sin(K_\rho-P_0)}{R_1}$$
$$\frac{\sin(P_2-P_0)}{V_\rho\cdot(t_2-t_0)}=\frac{\sin(K_\rho-P_2)}{R_1} \qquad \text{(A1.1)}$$

where P_i is the object's bearing at time point t_i;

R_1 is the distance to the object at time point t_1;

V_ρ, K_ρ are the relative velocity and heading of the object.

We divide the first equation by the second one:

$$\frac{\sin(P_1-P_0)\cdot(t_2-t_0)}{\sin(P_2-P_0)\cdot(t_1-t_0)}=\frac{\sin(K\rho-P_0)}{\sin(K\rho-P_2)} \qquad \text{(A1.2)}$$

By expanding the brackets and doing some simple developments, we obtain:

$$\begin{aligned}&\sin(P_1-P_0)\cdot(t_2-t_0)\cdot cos(P_2)\cdot tg(K\rho)-\\&-\sin(P_1-P_0)\cdot(t_2-t_0)\cdot\sin(P_2)=\\&=\sin(P_2-P_0)\cdot(t_1-t_0)\cdot cos(P_0)\cdot tg(K\rho)-\\&-\sin(P_2-P_0)\cdot(t_1-t_0)\cdot\sin(P_0),\end{aligned} \qquad \text{(A1.3)}$$

whence it follows that

$$K_\rho=arctg\frac{\sin(P_1-P_0)\cdot(t_2-t_0)\cdot\sin(P_2)-\sin(P_2-P_0)\cdot(t_1-t_0)\cdot\sin(P_0)}{\sin(P_1-P_0)\cdot(t_2-t_0)\cdot cos(P_2)-\sin(P_2-P_0)\cdot(t_1-t_0)\cdot cos(P_0)} \qquad \text{(A1.4)}$$

To increase the accuracy of estimation of the object's relative bearing, bearings P_0 and P_2 should have the maximum time diversity, and bearing P_1 should be approximately equal to the arithmetic mean of bearings P_0 and P_2.

To increase the accuracy of estimation of the object's relative bearing by using a larger number of measured bearings, it is necessary to form a set of equations (A1.4) for different time points and solve it by the least squares method.

Annex 2

FORMULA DERIVATION FOR CALCULATING THE RMS ERROR OF SIGNAL LEVEL MEASUREMENT

The signal power estimate is calculated by formula

$$\hat{W}_s = \hat{W}_{s+n} - \hat{W}_n \qquad \text{(A2.1)}$$

where $\hat{W}_s, \hat{W}_{s+n}, \hat{W}_n$ are the estimates of signal power, signal and interference mixture, and interference, respectively, in the frequency band Δf .

According to [23], the variance of estimate $\hat{W}$ of the signal power in the frequency band Δf is:

$$\sigma^2_{\hat{W}} = \frac{W^2}{\Delta f \cdot T} \qquad \text{(A2.2)}$$

where T is the time of signal accumulation during measurement.

Hence, taking into account (A2.1),

$$\sigma^2_{\hat{W}_s} = \sigma^2_{\hat{W}_{s+n}} + \sigma^2_{\hat{W}_n} = \frac{W^2_{s+n}}{\Delta f \cdot T} + \frac{W^2_n}{\Delta f \cdot T} = \frac{\left(W_s + W_n\right)^2 + W_n^2}{\Delta f \cdot T} \qquad \text{(A2.3)}$$

Relative variance of the signal power is

$$\sigma^2_{\hat{W}_s/otn} = \frac{\left(W_s + W_n\right)^2 + W_n^2}{W_s^2 \cdot \Delta f \cdot T} = \frac{\left(q+1\right)^2 + 1}{q^2 \cdot \Delta f \cdot T} \qquad \text{(A2.4)}$$

where $q = \frac{W_s}{W_n}$ is signal-to-noise ratio (by power) at the output of the linear part of the receiver.

Relative RMS error of the signal level estimate can be written as

$$\sigma_{\hat{W}_s/otn} = \frac{\sqrt{\left(q+1\right)^2 + 1}}{q \cdot \sqrt{\Delta f \cdot T}} \qquad \text{(A2.5)}$$

Relative RMS error of the signal level estimate in decibels can be presented as follows:

$$\sigma_{\hat{W}_s/otn} = 10 \cdot \lg\left(1 + \frac{\sqrt{\left(q+1\right)^2 + 1}}{q \cdot \sqrt{\Delta f \cdot T}}\right) \qquad \text{(A2.6)}$$